





C. U. SHAH UNIVERSITY WADHWAN CITY FACULTY OF SCIENCES

M.Sc.

MATHEMATICS SEM - IV

Syllabi (CBCS)



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Advanced Functional Analysis SUBJECT CODE: 5SC04AFA1

Teaching & Evaluation Scheme:-

Teaching hours/week Credit					Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total		Sessional Exam		University Exam		Internal		University	Total Marks
					Marks	Hrs	Marks	Hrs	Pr	TW		
4	0	0	4	4	30	1.5	70	3				100

Objectives:- The main objective of this course to learn

- Hilbert spaces (inner product, Cauchy-Schwarz inequality, norm, Caucy sequences, abstract Fourier expansions, the examples I² and L², direct sums, tensor products),
- Bounded and compact linear operators on Hilbert space (adjoint, projections, partial isometries, trace class, Hilbert-Schmidt class),
- Unbounded operators on Hilbert space, with applications to quantum theory (including Stone's Theorem) and ordinary differential equations (notably Sturm-Liouville problems),
- Spectral theory of bounded and unbounded self-adjoint operators on Hilbert space,

Prerequisites:-Basics of Analysis, Algebra and Topology.

Sr.	Course Contents
No.	
1	Inner product spaces, Polarization identity, Schwarz inequality, Parallelgram Law,
	orthonormal sets, Gram-Schmidt Orthonormalization, Hilbert spaces.
2	Approximation and optimization, Projections and Riesz representation theorems.
3	Bounded Operators on Hilbert Spaces: Bounded operators and adjoints, Normal, unitary
	and self-adjoint operators.
4	Spectrum and Numerical Range, Compact self-adjoint operators.



Learning Outcomes:-Students will be able to solve the problems related to content and they will be ready to learn advance courses on functional analysis.

Books Recommended:-

- 1. 'Functional Analysis', **Limaye, B. V.,** New Age International Publ. Ltd., New Delhi.
- 2. 'Real Analysis', Royden, H. L., Mc. Millan.

- 1. en.wikipedia.org/wiki/Functional analysis
- 2. www.umn.edu/~arnold/502.s97/functional.pdf
- 3. www.journals.elsevier.com/journal-of-functional-analysis/
- 4. http://www.uio.no/studier/emner/matnat/ifi/INF-MAT3360/index-eng.html
- 5. http://www.math.umn.edu/~garrett/m/fun/
- 6. http://www.nptel.iitm.ac.in/courses/111105037/



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Graph Theory SUBJECT CODE: 5SC04GRT1

Teaching & Evaluation Scheme:-

Teaching hours/week Credit					Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total			Sessional University Internal Uni		University	Total Marks			
					Marks	Hrs	Marks	Hrs	Pr	TW		
4	0	0	4	4	30	1.5	70	3				100

Objectives:-The main objective of this course is to learn concepts of graphs like tree, matrices of the graphs, chromatic numbers, Hamiltonian cycles, matching and covers.

Prerequisites:-Nothing special is required, any person familiar with basic of Mathematics can learn this subject.

Sr.	Course Contents
No.	
1	Quick view of basic terms about graphs: graph, vertex degree, paths, cycles, connected
	graph, tree, Euler graph, fundamental circuits, matrix representation of graphs.
2	Directed Graphs: definitions and examples, vertex degrees, some special types of digraphs, directed path and connectedness, Euler digraphs, trees with directed edges, spanning out-tree and spanning tree, fundamental circuits in digraphs, matrices A and B of digraphs, adjacency matrix of digraph.
3	Chromatic number, chromatic partitioning, chromatic polynomial, Four-color Problem, Hamiltonian cycles: necessary conditions, sufficient conditions, isomorphic graphs.
4	Matching and covers: maximum matching, Hall's matching condition, min-max theorems, independent sets, vertex cover, edge cover.



At the end of this course, the student should be able to apply the abstract concepts of graph theory in modelling and solving non-trivial problems in different fields of study.

Books Recommended:-

- 1. 'Graph Theory with applications to Engg. And Computer Science', Narsingh Deo, *Prentice Hall of India Pvt. Ltd., New Delhi*.
- 2. 'Introduction to Graph Theory', **Douglas, B. W.,** Pearson publication.
- 3. 'A first look at graph theory', Clark, J. and Holton, D. A., Allied Publishing Ltd.
- 4. 'Introduction to graph theory', **Robin, J. W.,** Pearson publication.

- 1. http://en.wikipedia.org/wiki/Graph theory
- 2. cr.yp.to/2005-261/bender2/GT.pdf
- 3. www.cs.columbia.edu/~sanders/graphtheory/
- 4. http://www.utm.edu/departments/math/graph/
- 5. http://www.personal.kent.edu/~rmuhamma/GraphTheory/graphTheory.htm
- 6. http://diestel-graph-theory.com/
- 7. http://www.graphtheory.com/



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Classical Mechanics SUBJECT CODE: 5SC04CLM1

Teaching & Evaluation Scheme:-

Teaching hours/week Credit					Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total		Sessional Exam		University Exam		Internal		University	Total Marks
					Marks	Hrs	Marks	Hrs	Pr	TW		
4	0	0	4	4	30	1.5	70	3				100

Objectives:-This course aims to provide a knowledge of Lagrangian formulation, Euler-Lagrange equation, Hamilton's variational principle, Hamilton's equation and canonical transformation.

Prerequisites:-

Basic knowledge of Mechanics, calculus etc.

Sr.	Course Contents
No.	
1	Lagrangian formulation: D'almbert's principle, principle of virtual work, classification of
	constraints, Lagrange's equation for holonomic systems, illustrations.
2	Euler-Lagrange equation, Hamilton's variational principle, derivation of Lagrange's
	equation from Hamilton's variational principle- generalized momentum-mechanics in
	configuration space, general conservation theorem, illustration.
3	Hamilton's canonical equation of motion, relation with Lagrange's equation,
	conservation theorems, variational principle approach to Hamilton's equation of
	motion, variational principle and Hamilton's equation, examples.
4	Canonical transformations, generating functions, symplectic conditions, infintesimal
	canonical transformations, examples. Poisson bracket formulation, general equation of
	motion and its formal solution, constants of motion, symmetry group.



After successful completion of this course, students will be able to

- Understand D'almbert's principle, principle of virtual work, Eularlangrange equation.
- Explain Hamilton's canonical equation of motion, Hamilton's variational principal.
- Find Lagrangian and Hamilton's equation of motion.
- Apply Lagrangian formulation, Hamilton's variational principal in practical situation.

Books Recommended:-

- 1. 'Classical Mechanics', **Goldstein, H., Poole, C. and Safko, J.,** *Pearson Education, Inc., Indian Low Price Edition, 2002.*
- 2. 'Classical Mechanics with Introduction to Nonlinear Oscillations and Chaos', **Bhatia**, *Narosa Publ. House*, 1997.

- 1. http://en.wikipedia.org/wiki/Classical mechanics
- 2. http://astrowww.phys.uvic.ca/~tatum/classmechs.html
- 3. http://nptel.ac.in/courses/115106068/
- 4. http://torassa.tripod.com/paper.htm
- 5. http://video.adm.ntnu.no/openVideo/serier/4fe2d4d3dcfd3



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Number Theory SUBJECT CODE: 5SC04NUT1

Teaching & Evaluation Scheme:-

Teaching hours/week Credit					Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total		Sessional Exam		University Exam		Internal		University	Total Marks
					Marks	Hrs	Marks	Hrs	Pr	TW		
4	0	0	4	4	30	1.5	70	3				100

Objectives:-The main objective of this course is to learn divisibility, congruence relations, power residues, quadratic reciprocity, diophantine equations, Number theoretic functions, continued fractions and rational approximation, partitions.

Prerequisites:-Knowledge of graduate level mathematics.

Sr.	Course Contents
No.	
1	Divisibility: foundations, division algorithm, greatest common divisor, Euclid's algorithm,
	Fundamental theorem, properties of primes.
	Arithmetical functions: the function [x], multiplicative functions, Euler's (totient)
	function $\varphi(n)$. The Mobius function $\mu(n)$, the functions $ au(n)$, and $\sigma(n)$, brief
	introduction of convolution of arithmetical functions, perfect numbers.
2	Congruences: definitions, Chinese Remainder theorem, the theorem of Fermat and
	Euler, Wilson's theorem, Lagrange's theorem, primitive roots, indices.
3	Miscellaneous topics: finite, infinite continued fractions, linear Diophantine equations
	x + by = c, Pell's equations, Pythagorean triplets, brief introduction of Fermat's last
	theorem.
4	Quadratic Fields: algebraic number fields, the quadratic fields, units, primes and
	factorization, Euclidean fields, the Gaussian field, Gaussian primes.



After successfully completion of this course, students will be able to

- Finding the greatest common divisor and least common multiple of a pair of natural numbers, and finding the linear form of the greatest common divisor.
- Prime factorization.
- Solving linear congruence and systems of simultaneous linear congruence.
- Understand the proofs of the theorems of Fermat, Wilson, and Euler.
- Primitive roots modulo primes and prime powers.
- Determining whether a quadratic congruence has solutions, and if so, finding them.

Books Recommended:-

- 1. 'An introduction to the Theory of Numbers', **Nivan, I., Zuckermann, H. S., Montgomery, H. L.**, *John Wiley & Sons Inc.*
- 2. 'Elementary number theory', **Burton, D. M.**, Universal Book stall, New Delhi.
- 3. 'A Concise introduction to the Theory of Numbers', **Baker, A.**, *Cambridge Uni. Press, Cambridge*.

- 1. en.wikipedia.org/wiki/Number theory
- 2. www.numbertheory.org/
- 3. http://mathworld.wolfram.com/NumberTheory.html
- 4. archives.math.utk.edu/topics/numberTheory.html
- 5. http://www.worldscientific.com/worldscinet/ijnt



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Advanced Algebra SUBJECT CODE: 5SC04ADA1

Teaching & Evaluation Scheme:-

Teaching hours/week Credit					Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total		Sessional Exam		University Exam		Internal		University	Total Marks
					Marks	Hrs	Marks	Hrs	Pr	TW		
4	0	0	4	4	30	1.5	70	3				100

Objectives:-This course aims to provide a knowledge of Modules, R-homomorphisms, first isomorphism theorem, annihilator, torsion element and torsion module etc.

Prerequisites:-

Knowledge of Group, ring etc.

Sr.	Course Contents
No.	
1	Modules, definition and examples, modules over a division ring, submodules and direct
	sums, products and quotient modules, R-homomorphisms, first isomorphism theorem,
	second isomorphism theorem, third isomorphism theorem.
2	Characterization of a module multiplication on an abelian group, factorization and lifting
	of module homomorphisms, correspondence theorem, annihilator, torsion element and
	torsion module, torsion free module.
3	External direct product and external direct sum, canonical projection, internal direct
	sum, direct summand, exact sequence, short exact sequence, split exact sequence. Free
	modules, cyclic modules, finitely generated free modules, invariant rank property and
	rank of a free module.



Infinite dimensional free modules, free modules over PIDs, invariant factor theorem for submodules, finitely generated modules over PIDs. Projective and injective modules, simple and semisimple modules, characterization of finitely generated semisimple modules, left and right semisimple rings and its applications to semisimple modules.

Learning Outcomes:-

After successful completion of this course, students will be able to

- Define modules and able to give different examples.
- Find External direct product and external direct sum.
- Explain terms exact sequence, short exact sequence, split exact sequence.
- Describe Infinite dimensional free modules, free modules over PIDs, invariant factor theorem for submodules.

Books Recommended:-

- 1. 'Algebra', Sahai, V. and Bist, V., Narosa Publishing House, New Delhi, 2008.
- 2. 'Basic Abstract Algebra', **Bhattacharya, P.B., Jain, S.K. and Nagpaul, S.R.,***Cambridge University Press, South Indian Edition 2002.*
- 3. 'Algebra Vol 3: Modules', **Luthar, I.S.** and **Passi, I.B.S.**, *Narosa Publishing House, New Delhi, 2004*.

- 1. http://www.math.uiuc.edu/~r-ash/Algebra/Chapter4.pdf
- 2. http://mathworld.wolfram.com/QuotientModule.html
- 3. http://www.math.iitb.ac.in/atm/final1.pdf
- 4. http://www.math.ntu.edu.tw/~jkchen/S04AA/S04AAL4.pdf



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Banach Algebras SUBJECT CODE: 5SC04BAA1

Teaching & Evaluation Scheme:-

Teaching hours/week Credit					Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total		Sessional Exam		University Exam		Internal		University	Total Marks
					Marks	Hrs	Marks	Hrs	Pr	TW		
4	0	0	4	4	30	1.5	70	3	1			100

Objectives:-This course aims to provide a knowledge of Banach algebra, regular and singular elements. Also it helps to understand Gel'fand Mazur theorem, Banach stone theorem. It will provide basic idea of C*algebra.

Prerequisites:-

Basic knowledge of Norm linear space, inner product space, algebra, topology and analysis.

Sr.	Course Contents
No.	
1	Banach algebras, examples, $C(X)$, $A(D)$, $C^1[0,1]$, $AC[0,1]$, $L^1(\mathbb{R})$. Regular and singular
	elements, topological divisors of zero, Gel'fand Mazur theorem.
2	Spectrum of an element and spectral radius, radical and semi-simplicity. Complex
	homomorphisms and maximal ideals.
3	The Gel'fand space, Gel'fand transform, Gel'fand representations of some concrete
	algebras. The Banach algebra $C(X)$, closed ideals of $C(X)$.
4	Banach-Stone theorem. InvolutiveBanach algebras, C*-algebras, Gel'fand-Naimark
	theorem for commutative C*-algebras.



After successful completion of this course, students will be able to

- Explain Banach algebra and different examples of Banach algebra.
- Understand Spectrum of an element and spectral radius, Complex homomorphisms.
- Use Gel'fand space, Gel'fand transform, Banach-Stone theorem in heigher mathematics like B*-algebras, C*-algebras.

Books Recommended:-

- 1. 'Introduction to Topology & Modern Analysis', **Simmons, G. F.,** *McGraw-Hill Book Company, Inc. 1963.*
- 2. 'A Course in Commutative Banach Algebras', Kaniuth, E., Springer, New York, 2009.
- 3. 'Banach Algebras', Larsen, R., Marcell-Dekker, 1973.
- 4. 'Automatic Continuity', Dales, H.G., Cambridge, 2000.

- 1. http://en.wikipedia.org/wiki/Banach algebra
- 2. http://homepage.ntlworld.com/ivan.wilde/notes/calg/calg.pdf
- 3. http://profsite.um.ac.ir/~moslehian/cfa/BA.HTM
- 4. http://mathworld.wolfram.com/BanachAlgebra.html
- 5. http://qchu.wordpress.com/2012/07/17/banach-algebras-the-gelfand-representation-and-the-commutative-gelfand-naimark-theorem/



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Mathematical Methods-II SUBJECT CODE: 5SC04MAM1

Teaching & Evaluation Scheme:-

Tea	aching	hours	/week	Credit	Evaluation Scheme/semester							
				Theory				Practical				
Th	Tu	Pr	Total		Sessio Exar		Univer: Exan	•	Internal		University	Total Marks
					Marks	Hrs	Marks	Hrs	Pr TW			
4	0	0	4	4	30	1.5	70	3				100

Objectives:-This course aims to provide knowledge of Euler's equation, geodesics, Integral equations, integro-differential equation, Fredholm integral equations and Bessel's equation.

Prerequisites:-

Basic knowledge of ordinary, partial differential equation, Laplace transform.

Sr.	Course Contents
No.	
1	Functionals, Euler's equation, other forms of Euler's equation, some special forms of
	Euler's equation, geodesics. Isoperimetric problems, several dependent variables,
	functionals involving higher order derivatives
2	Integral equations, types of integral equations, conversion of differential equation into
	an integral equation and vice versa, solution of integral equation, Integral equations of
	convolution type, Abel's integral equations, integro-differential equation
3	Fredholm integral equations, Fredholm alternative theorem, solutions of Fredholm
	integral equations for separable kernels, moving boundries.



Bessel's equation, Laguerre's equation, Hermite equation, Sturm-Liouville equations,
Conversion of various types of differential equations into Sturm-Liouville equation, their solutions

Learning Outcomes:-

After successful completion of this course, students will be able to

- Understand Euler's equation and it's different and special form.
- Explain integral equation and its different types.
- Explain Fredholm integral equations and give solution of Fredholm integral equations for separable kernels.
- Convert various types of differential equations into Sturm-Liouville equation and find its solution.

Books Recommended:-

- 1. 'Higher Engineering Mathematics', Grewal, B. S., Khanna Publs, 3rd Edition, Delhi.
- 2. 'An elementary course on variational problems in calculus', **Kumar, N.,** *Narosa publishing House, New Delhi, 2005*.
- 3. 'Calculus of variations with applications', **Gupta, A. S.,** *Prentice-Hall of India, New Delhi,* 1999.
- 4. 'Integral Equations and Applications', Mikhlin, S. G., Macmillan, 1964.

- 1. http://en.wikipedia.org/wiki/Integral equation
- 2. http://www.academia.edu/1741456/Compact Operators
- 3. http://en.wikipedia.org/wiki/Fredholm integral equation
- 4. https://people.math.osu.edu/gerlach.1/math/BVtypset/node171.html
- 5. http://www.iitg.ernet.in/physics/fac/charu/courses/ph402/SturmLiouville.pdf



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Operator Theory SUBJECT CODE: 5SC04OPT1

Teaching & Evaluation Scheme:-

Tea	aching	hours	/week	Credit	Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total		Sessional University Exam Exam Internal		University	Total Marks				
					Marks	Hrs	Marks	Hrs	Pr TW			
4	0	0	4	4	30	1.5	70	3				100

Objectives:-This course aims to provide a knowledge of orthogonal complement in H, bounded operator, Fuglede-Putnam-Rosenblum theorem, spectral theorem and its applications, Hilbert-Schmidt operators and their properties etc.

Prerequisites:-

Basic knowledge of Hilbert space, inner product space, metric space etc

Sr.	Course Contents
No.	
1	Review of Hilbert space H, orthogonal complement in H, bounded operator, existence of
	adjoint operator and its properties, self-adjoint operator and its properties, unitary
	operator and its properties, Fuglede-Putnam-Rosenblum theorem (i.e., Commutativity
	Theorem)
2	Resulution of the identity E, the algebra $L^{\infty}(E)$, identifying $L^{\infty}(E)$ with a closed
	subalgebra of $BL(H)$, spectral theorem and its applications, spectral decomposition
3	Symbolic calculus for normal operators and applications, invariant subspace problem,
	eigenvalues of normal operators, positive operators and square roots, polar
	decomposition and its uniqueness, unitarily equivalent
4	Hilbert-Schmidt operators and their properties, trace class operators, Hilbert-Schmidt
	and trace class norms, relations between these two types of operators.



After successful completion of this course, students will be able to

- Understand orthogonal complement in H, bounded operator, existence of adjoint operator, self-adjoint operator, Commutativity Theorem.
- Give properties of existence of adjoint operator, unitary operator, self-adjoint operator.
- Explain Resulution of the identity E, the algebra $L^{\infty}(E)$, identifying $L^{\infty}(E)$ with a closed subalgebra of BL(H).
- Find application of spectral theorem, relations betweenHilbert-Schmidt operators and trace class.

Books Recommended:-

- 1. 'Functional Analysis', Rudin, W., Tata McGraw Hill Pub. Company, New Delhi, 1973.
- 2. 'A Course in Operator Theory', **Conway, J. B.,** *Graduate Studies in Mathematics, Volume 21, American Mathematical Society, Rhode Island, 2000.*

- 1. http://www.math.ksu.edu/~nagy/real-an/2-07-op-th.pdf
- 2. https://www.math.ucdavis.edu/~hunter/book/ch8.pdf
- 3. http://en.wikipedia.org/wiki/Invariant_subspace_problem
- 4. http://mathworld.wolfram.com/Hilbert-SchmidtNorm.html



DEPARTMENT OF MATHEMATICS

COURSE: M.Sc. SEMESTER: IV SUBJECT NAME: Problem Solving-III SUBJECT CODE: 5SC04PRS1

Teaching & Evaluation Scheme:-

Teaching hours/week				Credit	Evaluation Scheme/semester							
					Theory				Practical			
Th	Tu	Pr	Total		Sessional University Exam Exam		Internal University		Total Marks			
					Marks	Hrs	Marks	Hrs	Pr TW			
4	0	0	4	4	30	1.5	70	3				100

Objectives:-Objective of this course is to develop skills of example solving.

Prerequisites:-

Knowledge of particular topics up to bachelor or master level.

Sr.	Course Contents
No.	
1	Real Analysis: : Only Problems based on the following topics, Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral.
2	Number Theory: Only Problems based on the following topics, Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements, Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots.



Topology: Only Problems based on the following topics, basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

Learning Outcomes:-After successful completion of this course students will be able to solve examples of respective topics. And hence it will be useful to them for NET/SET Examination.

Books Recommended:-

- 1. 'UGC CSIR NET/SET (JRF & LS) Mathematical Sciences', **Sharma, P., Sharma, N., Singh, S.,** *Arihant Publications (India) Limited.*
- 2. 'Mathematical Analysis', **Malik, S.C., Arora, S.,** New age international(P) limited publishers.
- 3. 'Real Analysis', Royden, H.L., Mc. Millan.
- 4. 'Topology: A First Course', Munkres, J., Prentice Hall of India Pvt. Ltd., New Delhi.
- 5. 'Elementary number theory', Burton, D. M., Universal Book stall, New Delhi.

- 1. http://en.wikibooks.org/wiki/Real Analysis
- 2. http://mathworld.wolfram.com/Topology.html
- 3. www.journals.elsevier.com/topology/
- 4. www.numbertheory.org/